When interpreting for non-experts

For the 2021 reprinting of *Gravitation* I wish to add two comments about black holes. One is to emphasize Finkelstein's contribution more strongly. The second is to offer thoughts on the significance of the spacetime metrics for black holes (BHs) as interpreted for non-experts.

David R. Finkelstein's "unidirectional membrane" paper (Finkelstein 1958) was important for the timely recognition of what would later be called "the black hole horizon." In fact, it was so important that he should have been considered for a share in the 2020 Nobel Prize in Physics had he lived that long. Only after this Finkelstein paper could one imagine that *a dynamic empty spacetime* exists inside the BH horizon where it lies beyond the collapsing matter – matter which may have promoted the production of the black hole.

The metric form I now think should be called simply the "Finkelstein metric" [Eq. (2) of Box 31.2] arose originally in the 1922 gravitation theory proposed by A. N. Whitehead. Arthur Eddington showed why *Whitehead's metric* for the field of the Sun gave the same results (e.g., for the perihelion of Mercury) as Einstein's solar metric: He found that the Whitehead metric satisfied the Einstein field equations. But neither Whitehead nor Eddington expected this metric to be valid at $r = 2M$, since it was to be used only in the empty space outside the Sun or other large static spherical object. Neither one remarks on the regularity of the metric at $r = 2M$.

My second comment concerns how a black hole can best be described to less technical audiences and primarily concerns what plausibly happens inside a black hole horizon. For this purpose, it is a mistake to use the Kruskal metric or a Penrose conformal diagram. Each of these hides the Killing symmetry that generates invariance under time translation outside the horizon and a smoothly fitting spatial translation symmetry inside the horizon. (Because light cones in Kruskal coordinates are easier to sketch—they are all 45-degree straight lines—than the light cones in Finkelstein coordinates, I mostly used the Kruskal metric at the time our book was evolving.) In this book, the most useful BH sketch using Finkelstein coordinates is the first figure in Box 31.2, section A, p. 829, where the region below $\tilde{V} = 0$ should be imagined to contain the field of collapsing matter. More beautifully drawn corresponding figures can be found in Penrose's early papers on BHs. Non-experts will not want to study these diagrams, but writers who want to describe BHs to non-experts should study them and specifically note the huge spatial extent found inside the BH.

To emphasize this unintuitive huge spatial extent inside a BH, consider the case where a small mass is dropped into a quiescent BH at one time, and then a million years later a similar small mass is added in the same way, that is, the world lines of the first and second drop-ins are related to each other by the time-translation symmetry of the BH metric. While the first and second world lines outside the horizon have a million-year-long proper time difference, the parts of their world lines inside the BH horizon are related by a constant-r separation *spatially* of about a million light-years. Thus, inside a stellar mass BH of a few kilometers in apparent diameter there lies a space-filled tube many light-years long with expanding spatial extent. The same remark applies also to the Kerr metric describing a spinning black hole. There this tube is bounded by the outer and inner constant r horizons but contains $r = M$ for all plausible spin values. Because the matter that fell in to form the BH is so distant (spatially, along lines of constant r, θ, φ) from any drone sent to survey it, I consider this matter to have been flushed out of our universe. The geometry of spacetime inside the BH horizon is so weird and unfamiliar that I think it improper to suggest that matter there is merely compressed infinitely within the BH.

Einstein's gravity theory appears to me quite plausible in all its descriptions inside black holes to a minor depth of about half the coordinate size of the BH. That is, I expect new physics is necessary only for $r < M$ in its description of quiescent black holes. The reason for accepting it at and near the BH horizon is that spacetime is little curved at the horizon.[1] Much deeper at $r < M$ there are unsolved problems that appear in the Kerr metric. Most BHs are likely to be rotating, and the Kerr metric includes a place not much below $r = M$ where the metric is so obtuse (Cauchy horizon) that the theory seems to recuse itself from giving any robust evolution prescription for the collapsing BH spacetime. All current grand theories of fundamental physics seem adequate on microscales and very high energies not approaching the Planck mass or size. The most evidence for ignorance now seems to be only for matter seen at extremely low densities—dark matter and dark energy. So I raise the question whether two points are related: (1) the failure of Kerr metrics to appear stable deep inside black holes, and (2) our need to understand low-energy densities seen observationally only in huge objects such as galaxies and

[1] Riemann tensor invariants are about M/r^3 in the Kerr metric, thus 1/M^2 a bit inside the horizon at $r = M$. Corrections to flat space needed for geodesics to curve relative to each other then occur for distances M in space with this much curvature. For an astronaut in low orbits around the Earth, the curvature causes parallel "free fall" world lines to cross in about 45 minutes. For black holes the distance for spacelike geodesics to bend is about 5 light-hours in a 20 billion solar mass BH inside its horizon at about $r = M$. This is a far weaker tidal force than the astronaut feels inside a low Earth orbit spacecraft.

their clusters.

Summary

The matter that collapses to form a black hole plays only an enzymatic role. It assists a region of spacetime to organize itself into a new, stable region of otherwise empty spacetime and is then flushed out of our universe. There are huge amounts of space, but little time, inside a black hole. Improvements to general relativity are needed, but primarily in the treatment of very large, very low-density regions such as $r < M$ in BHs.

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