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Introduction

LETTER FROM A FRIEND

"I JUST read the little blurb about you in the *Swarthmore College Bulletin,*" the email began. "To say it's been awhile since we last spoke would be an understatement. Your career has certainly been an interesting and winding one.

"The reason I'm contacting you is to see if you can offer me some guidance/help on solving a math problem. . . . I've written up my results so far which I think are very interesting, and have hypothesized a solution. I'd like to share it with you if you have an interest.

"Let's catch up."

The email was signed Howard Stern, and I guess the first thing I have to say is no, it was not *that* Howard Stern. *This* Howard Stern was a classmate of mine in college, long before a certain radio personality made the name famous (or infamous). He was a double major in math and economics, and I was a major in math. We were both friends and rivals. After graduation, though, we fell out of contact: he went to graduate school in economics at MIT, while I followed the siren song of pure mathematics to Princeton University. When his email arrived out of the blue, I had not spoken a word to him in 33 years.

Howard explained that he had made up the problem when he was a graduate student but could not solve it. He never lost his curiosity, though. Over the next 25 years, he had posed it to various mathematicians, thinking that one of them must know the answer already. But none of them did. Moreover, to his increasing frustration, none of them seriously engaged with the problem.

But when he wrote to me, he finally found a sympathetic ear. From the moment I started thinking about Howard's problem, I couldn't take my mind off it. I spent the next 12 months working on it whenever I had a free hour or two.

At the time, I felt guilty about all the time I was devoting to Howard's problem. I do have a job, after all. I'm a freelance writer, and time spent on math puzzles is time that I'm not earning money. But I could not resist. It had everything a math problem should have. I thought I had it licked more than once, but every time it "fought back" and I had to go back to the drawing board. But it never seemed impossibly hard. An impossibly hard problem is one you have no idea how to solve. With Howard's problem, I had plenty of ideas, and the question was which one would work (maybe all of them combined?).

Howard's problem was the best gift one mathematician can give to another: a puzzle. "Here's something I can't figure out; what do you think?" But completely unexpectedly, it became for me a deeper sort of gift. It became a chance to rewind time and reconnect to a younger version of myself, a version I thought was lost.

Let me explain. Once upon a time, I had loved mathematics. My father had encouraged me as a kid, posing problems on permutations and series that were way beyond my grade level. I had lapped up the books of Martin Gardner, whose "Mathematical Games" column in *Scientific American* awakened many readers to the fun side of mathematics.

In college, my infatuation deepened. I discovered that math is more than a compendium of fun facts or cool tricks. It is a way of thinking. In mathematics, every statement has to be proved or at least provable. It cannot be faked or fudged. The best proofs, the ones that bring you closest to the truth of the matter, are also the most beautiful (or "elegant," as we like to say). Mathematics seemed to me like the last refuge for truth and beauty, in an era when truth has become malleable and poets have abandoned beauty.

I admit that my view of mathematics was hyperromantic, and like many romances it faded, bit by bit. Graduate school was a struggle. For the first time I met people who were better at math than me. I listened to lectures I could not understand. Fortunately, I had a mentor and dissertation adviser who told me not to worry about any of that. What I really needed, Fred Almgren said, was not to take more classes but to get started on a problem. Only then would I find out what I needed to know, and most likely I would have to teach it to myself.

You'll see that approach to mathematics in this book. I would say it is the way of mathematics research, but "research" is a heavy word, laden with the implication that you have to be a professional mathematician to do it. Throughout this book, I will use a term that Francis Su introduced in his book *Mathematics for Human Flourishing*: "math explorer." Exploring math is something anybody can do.

In some ways, math exploration is the exact opposite of classroom mathematics. As a math student, you learn a multitude of facts and methods, and it's not clear what you will need them for. But as a math explorer, everything you learn is directed by the problem you are working on. If you discover that you need to know, for instance, the Berry–Esseen Theorem, then it doesn't matter that you have never heard of it before. You go out and find a book, or a colleague, or even somebody on the internet who can explain it to you.

Also, the time frame of math exploration is measured not in minutes but in weeks, months, or years. This is again an unfamiliar experience for anyone who has received a conventional math education. School mathematics is all about quickness, all about knowing the answer—or at least, knowing how to find it—even before you start. This myth is immensely frustrating to students who don't get things instantly. They absorb this poisonous message: If you can't see the answer in 30 seconds, you're no good at mathematics. After hearing this message enough times, they give up.

How many of these students would stick around if they knew that mathematics was like a poem, one that is never in a hurry to reveal its true meaning? What would they think if somebody told them that research mathematicians spend most of their time working on problems that they have *no clue* how to solve? If we knew how to solve it, then someone else would have done it first. To solve such a problem, you need to be patient and slow.

A beautiful role model for slow mathematics is June Huh, one of the four recipients of the Fields Medal in 2022. (The Fields Medal is widely considered the most prestigious award in math, a sort of equivalent of the Nobel Prize.) In an article for *Quanta Magazine*, journalist Jordana Cepelewicz interviewed many mathematicians who all agreed: Huh is amazingly slow! You wouldn't think he could have passed even first-year calculus. He spends so much time on even the most elementary arguments. In one case, he and two collaborators had finished a proof, but Huh wasn't satisfied with it. He didn't think it got to the heart of the problem. They spent *two years* reworking the proof until it satisfied Huh—and all three agreed that that the new solution was better than the first one. In mathematics, slow and steady can absolutely win the race. Perhaps not every time, but much more often than our educational system built for jackrabbits would make you believe.

I have to tell you another reality: Math exploration can be extremely frustrating. You might have to try five ideas that don't work before you find one that does. Sometimes you just have to put the problem aside and come back to it later. But if you're patient enough and determined enough, occasionally you'll get a reward. Suddenly you see how everything fits together.

In 13 years as a professional mathematician, I experienced maybe two or three such epiphanies. They always gave me a profound sense of awe and privilege, as if for a moment I had caught a glimpse of the blueprint for the universe.

But by the time I received Howard's email, that era of my life was long past. After 13 years, academia spat me out, and thus began the "interesting and winding" journey that Howard had alluded to. The collapse of my mathematical career freed me to pursue my passion for writing, which had been with me even longer than my passion for mathematics. With much help from friends and mentors, I reinvented myself as a science journalist and no longer thought of myself as a mathematician.

Only my wife would scold me sometimes, “Don’t say that! You’re still a mathematician, but you’re just doing something else for a living.”

Howard’s problem proved that Kay was right. It returned to me all the things I loved about mathematics, with almost none of the agony. For the first time in many years, I found myself once again inhabiting a familiar space, working on a mathematics problem. It is like reading an enthralling mystery—but even better, because the universe itself writes the script. You don’t know how or even whether the story will end. To be honest, when I first started working on Howard’s problem, I didn’t think there was an end. I just wanted to see what I could figure out. Even as I got close to a solution, I wondered: What will be the catch? Where will the insurmountable obstacle appear?

Of course, I am now a storyteller by trade, so I am not going to give away the end yet. But already I can say what my reasons are for writing this book. I’d like to write a how-to book for math explorers. Alas, such a book is impossible, because every problem will be different. But I can give you some useful guidelines: Do examples. Ask questions. Allow yourself to wander, not march toward a prescribed destination. Look for the clues left by your fellow explorers before you. Look for patterns. Venture wild and crazy guesses and then subject them to cold and sober scrutiny.

By telling my story, and showing you how these guidelines worked for me, I hope I can inspire you and empower you to try them too.

The second reason for writing this story is that it’s the only math story that is mine to tell. As a science writer, I spend my time writing about other people’s ideas and discoveries, and I love doing that. But at best, I’m only holding up a mirror to their work. First person is still the best, the rawest, and the most intimate voice for a writer: it connects *me* to *you*, with no intermediaries. So, call me selfish. I want to write a first-person math story just once.

Finally, I want to tell a story that every reader can relate to, whether you know anything about math or not. It’s the story of reuniting with a lost love. I hope it will help you understand why math is so special to the people who fall under its spell. And I hope my story encourages you to rediscover and revive your own long-smoldering passion, whatever it may be, and no matter how odd or unusual it may appear to others.

Before we start, I need to make one further introduction. Although math is a profoundly creative endeavor, it is also extraordinarily difficult to think a truly new thought or ask a truly new question. No matter how narrow the path, no matter how obscure the question, if you look hard enough, you will always find the footsteps of someone who has gone before you. Everyone who has walked your path before is like a hidden collaborator.

So it should come as no surprise to find out that Howard's problem was not entirely new. To find the first footsteps on this path, we have to go back more than 2,000 years, to a legendary figure in Chinese history. Allow me to introduce you . . . to Master Sun.

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