CONTENTS

1

The Confluence of Nature and Mathematical Modeling

> What distinguishes a mathematical model from, say, a poem, a song, a portrait or any other kind of "model," is that the mathematical model is an image or picture of reality painted with logical symbols instead of with words, sounds or watercolors.
>
> —JOHN L. CASTI

(i) Confluence

Over the many years since my heart surgery in 1996, as I have walked daily to and from my university office, I have continued to train myself to observe the sky, the birds, butterflies, trees, and flowers, something I had not done earlier in a conscious way (although I did watch out for fast-moving cars and unfriendly dogs). Despite living in suburbia, I found, and continue to find, that there are many wonderful things to see: clouds exhibiting wave-like patterns; splotches of colored light some 22 degrees away from the Sun (sundogs, or parhelia), wave after wave of Canada geese in "vee" formation; waves (and a following region of calm water) spreading out on the surface of a puddle as a raindrop spoils its smooth surface; the occasional rainbow arc; even the iridescence on the neck of those rather annoying birds, pigeons; and many, many more nature-given delights. And I was never late for my first class of the morning (though it was close sometimes!).

As noted briefly above, the idea for this book was driven by a fascination on my part for the way in which so many of the beautiful phenomena observable in the natural realm around us can be described in mathematical terms (at least in principle). What are some of these phenomena? Some have been already mentioned in the preface, but for a more complete list we might consider rainbows, glories, halos (all atmospheric occurrences), waves in air, on

Earth, and in oceans, rivers, lakes, and puddles (made by wind, ship, or duck), cloud formations (e.g., billows, lee waves), tree and leaf branching patterns (including phyllotaxis), the proportions of trees, the wind in the trees, mud-crack patterns, butterfly markings, leopard spots, and tiger stripes; in short, if you can see it outside, and a human didn't make it, it's probably described in here! That, of course, is an exaggeration, but this book does attempt to answer on varied levels the fundamental question: what kind of scientific and mathematical principles undergird these patterns or regularities that I claim are so ubiquitous in nature?

Two of the most fundamental and widespread phenomena that occur in the realm of nature are (i) the scattering of light and (ii) wave motion. Both may occur almost anywhere given the right circumstances, and both may be described in mathematical terms at varying levels of complexity. It is, for example, light scattered by both air molecules and the much larger dust particles (or, more generally, aerosols) that combine to give the amazing range of colors, hues, and tints at sunrise or sunset that give us so much pleasure. The deep blue sky above and the red glow near the Sun at the end of the day are due to molecular scattering of light, and dust or volcanic ash can render the glow quite spectacular at times.

The rainbow is formed by sunlight scattered in preferential directions by near-spherical raindrops: scattering in this context means refraction and reflection (although there many other fascinating features of light scattering that will not be discussed in great detail here). Using a simple mathematical description of this phenomenon, Descartes in 1637 was able to "hang the rainbow in the sky" (i.e., deduce its location relative to the Sun and observer), but to "paint" the rainbow required the genius of Newton some thirty years later. The bright primary and fainter secondary bows are well described by elementary mathematics, but the more subtle observable features require some of the most sophisticated techniques of mathematical physics to explain them. A related phenomenon is that of the "glory", which is a set of colored, concentric rainbow-like rings surrounding, for example, the shadow of an airplane on a cloud below it. This, like the rainbow, is also a "backscatter" effect, and, intriguingly, both the rainbow and the glory have their counterparts in atomic and nuclear physics; mathematics is a unifying feature between these two widely differing contexts. The beautiful (and commonly circular) arcs known as halos, no doubt seen best in arctic climes, are formed by the refraction of sunlight through ice crystals of various shapes in the upper atmosphere. Sundogs, those colored splashes of light often seen on both sides of the Sun when high cirrus clouds are present, are similarly formed.

Like the scattering of light, wave motion is ubiquitous, though we cannot always see it directly. It is manifested in the atmosphere, for example, by billow clouds and lee wave clouds downwind from a hill or a mountain. Waves on the surface of puddles, ponds, lakes, and oceans are governed by mathematical relationships between their speed, their wavelength, and the depth of the water. The wakes produced by ships or ducks generate strikingly similar patterns relative to their size; again this correspondence is described by mathematical expressions of the physical laws that govern the motion. The situation is even more complex in the atmosphere: the "compressible" nature of a gas renders other types of wave motion possible. Sand dunes are another complex and beautiful example of waves. They can occur on a scale of centimeters to kilometers, and, like surface waves on bodies of water, it is only the waveform that actually moves; the body of sand is stationary (except at the surface).

In the plant world, the arrangement of leaves around a stem, or of seeds in a sunflower or a daisy face, shows, in the words of one mathematician (H.S.M. Coxeter), "a fascinatingly prevalent tendency" to form recurring numerical patterns, studied since medieval times. Indeed, many of these patterns are intimately linked with the "golden number" $((1+\sqrt{5})/2 \approx 1.618)$ so beloved of Greek mathematicians long ago. The spiral arrangement of seeds in the daisy head is similarly (but not identically) replicated in the sweeping curve of the chambered nautilus shell, or its helical counterpart, the cerithium fasciatum (a thin pointy shell). The curl of a drying fern and the rolled-up tail of a chameleon exhibit types of spiral arcs, which in Chapter 11 we shall identify as "metallic spirals."

In the animal kingdom, coat patterns (e.g., on leopards, cheetahs, tigers, and giraffes) and wing markings (e.g., on butterflies and moths) can be studied using mathematics, specifically by means of the properties and solutions of so-called reaction-diffusion equations (and also other types of mathematical models). The reaction-diffusion equations describe the interactions between chemicals ("activators" and "inhibitors") that, depending on the conditions, may produce spots, stripes, or more "splodgy" patterns. There are fascinating mathematical problems involved in this subject area, and also links with topics such as patterns on fish (e.g., angel fish) and on seashells. In view of earlier comments, seashells combine the effects of both geometry and pattern formation mechanisms, and mathematical models can reproduce the essential features observed in many seashells.

Cracks also, whether formed in drying mud, tree bark, or rapidly cooling rock, have their own distinctive mathematical patterns; frequently they are hexagonal in nature. River meanders, far from being "accidents" of nature, define a form in which the river does the least work in turning (according to one class of models), which then defines the most probable form a river can take: no river, regardless of size, runs straight for more than ten times its average width.

Many other authors have written about these patterns in nature. Ian Stewart has noted in his popular book *Nature's Numbers* (1995) that "we live in a universe of patterns. . . . No two snowflakes appear to be the same, but all possess six-fold symmetry." Furthermore, he states that "there is a formal system of thought for recognizing, classifying and exploiting patterns. . . . It is called mathematics. Mathematics helps us to organize and systemize our ideas about patterns; in so doing, not only can we admire and enjoy these patterns, but also we can use them to infer some of the underlying principles that govern the world of nature. . . . There is much beauty in nature's clues, and we can all recognize it without any mathematical training. There is beauty too in the mathematical stories that . . . deduce the underlying rules and regularities, but it is a different kind of beauty, applying to ideas rather than things. Mathematics is to nature as Sherlock Holmes is to evidence."

We may go further by asking questions like those posed by Peter S. Stevens in his lovely book *Patterns in Nature* (1974). He asks, "Why does nature appear to use only a few fundamental forms in so many different contexts? Why does the branching of trees resemble that of arteries and rivers? Why do crystal grains look like soap bubbles and the plates of a tortoise shell? Why do some fronds and fern tips look like spiral galaxies and hurricanes? Why do meandering rivers and meandering snakes look like the loop patterns in cables? Why do cracks in mud and markings on a giraffe arrange themselves like films in a froth of bubbles?" He concludes in part that "among nature's darlings are spirals, meanders, branchings, hexagons, and 137.5 degree angles. . . . Nature's productions are shoestring operations, encumbered by the constraints of three-dimensional space, the necessary relations among the size of things, and an eccentric sense of frugality."

In the book *By Nature's Design* (1993), Pat Murphy expresses similar sentiments, writing, "Nature, in its elegance and economy, often repeats certain forms and patterns . . . like the similarity between the spiral pattern in the heart of a daisy and the spiral of a seashell, or the resemblance between the branching pattern of a river and the branching pattern of a tree, . . . ripples that flowing water leaves in the mud, . . . the tracings of veins in an autumn leaf, . . . the intricate cracking of tree bark, . . . the colorful splashings of lichen on a boulder. . . . The first step to understanding—and one of the most difficult—is to see clearly. . . . Nature modifies and adapts these basic patterns as needed, shaping them to the demands of a dynamic environment. But underlying all the modifications and adaptations is a hidden unity. Nature invariably seeks to accomplish the most with the least—the tightest fit, the shortest path, the least energy expended. Once you begin to see these basic patterns, don't be surprised if your view of the natural world undergoes a subtle shift."

Another fundamental (and philosophical) question has been asked by many: How can it be that mathematics, a product of human thought independent of experience, is so admirably adapted to the objects of reality? This fascinating question I do not address here; let it suffice to note that, hundreds of years ago, Galileo Galilei stated that "[The Universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language." Mathematics is certainly the language of science, but it is far, far more than a mere tool, however valuable, for it is of course both a subject and a language in its own right. But lest any of us balk at the apparent need for speaking a modicum of that language in order to more fully appreciate this book, the following reassuring statement from Albert Einstein, written to a young admirer in junior high school, should be an encouragement. He wrote, "Do not worry about your difficulties in mathematics. I can assure you that mine are still greater." Obviously anyone, even scientists of great genius, can have difficulties in mathematics (one might add that it's all a matter of relativity in this regard).

Obviously a significant component of this book is the application of elementary mathematics to the natural world around us. As I have tried to show already, there are many mathematical patterns in the natural world which are accessible to us if we keep our eyes and ears open; indeed, the act of "asking questions of nature" can lead to many fascinating "thought trails," even if we don't always come up with the correct answers. First, though, let me remind you (unnecessarily, I am sure) that no one has all the answers to such questions. This is true for me at all times of course (and not just as a parent, a grandparent, and a professor), but especially so in a subject as all-encompassing as 'mathematics in nature,' because there will always be "displays" or phenomena in nature that any given individual will be unable to explain to the satisfaction of everyone, for the simple reason that none of us is ever in possession of all the relevant facts, physical intuition, mathematical techniques, or other requirements to do justice to the observed event. However, this does not mean that we cannot appreciate the broad principles that are exemplified in a rainbow, lenticular cloud, river meander, mud crack, or animal pattern. Most certainly we can.

It is these broad principles—undergirded by mathematics, much of it quite elementary—that I want us to perceive in a book of this admittedly rather free-ranging nature. My desire is that, by asking mathematical questions of the phenomena, we will gain (i) some understanding of the symbiosis that exists between the basic scientific principles involved and their mathematical description, and (ii) a deeper appreciation for the phenomenon itself, its beauty (obviously rather subjective) and its relationship to other events in the natural world around us. I have always found, for example, that my appreciation for a rainbow is greatly enhanced by my understanding of the mathematics and physics that undergird it (and, by the way, some of the mathematics can be extremely advanced; some references to this literature are provided in the bibliography). It is important to remember that this is a book on aspects of *applied mathematics*, and there will be at times some more advanced and even occasionally rigorous mathematics (in the form of theorems and sometimes proofs); for the most part however, the writing style is intended to be informal. And so now, on to the topic of. . .

(ii) . . . Modeling

Another point to make is that I have deliberately chosen to emphasize *continuum* mathematical models, as opposed to *discrete* ones (though you *will* encounter several difference equation models and a little epidemic modeling using transition matrices). This in effect means differentiable (and hence continuous) functions make their appearance, as is necessary for the application of differential calculus. And while many natural (and other) processes are *not* continuous, a great many can be modeled as such, at least to a first approximation.

An obvious question to be asked at the outset is: *what is a mathematical model?* One basic answer is that it is the formulation in mathematical terms of the assumptions and their consequences believed to underlie a particular real-world problem. The aim of mathematical modeling is the practical application of mathematics to help unravel the underlying mechanisms involved in, for example, economic, physical, biological, or other systems and processes. Common pitfalls include the indiscriminate, näive, or uninformed use of models, but when developed and interpreted thoughtfully, mathematical models can: (i) provide insight into the nature of the problem; (ii) be useful in interpreting data; and (iii) stimulate experiments. There is not necessarily a "right" model, and obtaining results which are consistent with observations is only a first step and does not imply that the model is the only one that applies, or even that it is "correct." Furthermore, mathematical descriptions are not explanations, and never on their own can they provide a complete solution to the biological (or other) problem—often there may be complementary levels of description possible within the particular scientific domain. Collaboration with scientists or engineers is needed for realism and help in modifying the model mechanisms to reflect the science more accurately. On the other hand, workers in nonmathematical subjects need to appreciate what mathematics (and its practitioners) can and cannot do. Inevitably, as always,

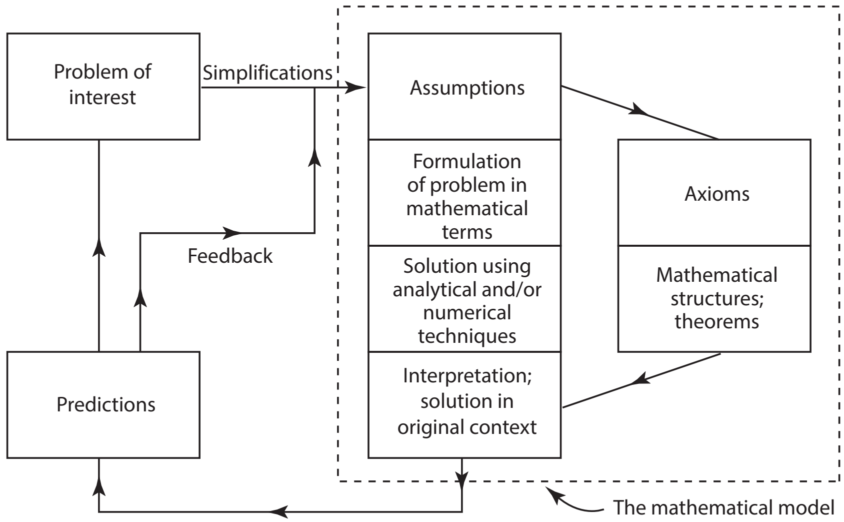

FIGURE 1.1. A generic flow chart illustrating stages and levels of mathematical modeling. Reprinted with permission of Birkhäuser, copyright 1997, from the book "A Survey of Models for Tumor-Immune System Dynamics," edited by J. A. Adam and N. Bellomo.

good communication between the interested parties is a necessary (but not sufficient) recipe for success. For further details, see Murray (1981).

In the preface mention was made of fundamental steps necessary in developing a mathematical model: formulating a real-world problem in mathematical terms using whatever appropriate simplifying assumptions may be necessary; solving the problem thus posed, or at least extracting sufficient information from it; and, finally, interpreting the solution in the context of the original problem (which, as noted above, may include validation of the model by testing both its consistency with known data and its predictive capability). Thus the art of good modeling relies on (i) a sound understanding and appreciation of the scientific or other problem; (ii) a realistic mathematical representation of the important phenomena; (iii) finding useful solutions, preferably quantitative ones; and (iv) interpretation of the mathematical results—insights, predictions, etc. Sometimes the mathematics used can be very simple. The usefulness of a mathematical model should not be judged by the sophistication of the mathematics, but by different (and no less demanding) criteria.

Although techniques of statistical analysis may frequently be used in portraying and interpreting data, it is important to note two fundamentally distinct approaches to mathematical modeling, which differ somewhat in both mathematical and philosophical characteristics. These are (i) deterministic models and (ii) probabilistic and statistical (sometimes referred to as stochastic) models. One's preference often depends upon the way one has been trained, which usually determines the way one looks at the world mathematically. Most of this book is written from the deterministic perspective. A general summary of the philosophy and methodology of this approach (with many references to applications in cancer biology) may be found in Chapter 2 of the book edited by Adam and Bellomo (1997). Deterministic models frequently possess the property that a system of interest (e.g., a population of cells, a mixture of chemicals or enzymes, a density or pressure imbalance in a gas) can be "observed" mathematically to evolve away from some initial configuration.

Frequently in deterministic models the assumption is made that the dependent variables are at least differentiable functions of their arguments, and hence are continuous. This assumption is quite reasonable when the magnitude of (say) the population of cells is very much larger than a typical change (increase or decrease) in the population. For a small tumor composed of about a billion cells, the individual cell cycles have become asynchronous, and so, on the basis of a simple model, the evolution of the tumor can be reasonably described using the techniques of differential and integral calculus. The situation is inevitably much more complex than this, and many other factors need to be included to derive a realistic model of tumor growth. Regardless of the complexity, deterministic models may be used to predict the size of the tumor as a function of time, in particular, to predict $N(t)$, the number of cells the tumor will have at time t. In practice, however, there is an element of uncertainty in every event from the growth of a tumor to catching a bus. External factors, usually beyond our control, play a role in determining the outcome of the event. Thus diet, fitness, treatment regimen, and mental attitude may contribute to the eventual outcome for the cancer patient, while a faulty alarm clock used by a bus driver may influence whether the bus arrives on time or not. Similarly, in studying the relation between stature and heredity, differences in environment and nutrition may be sources of uncertainty in the results. Stochastic models enable researchers to identify and study many of these uncertainties.

Note also that, in an experiment, errors that arise may be classified as random or systematic. The word *error* as used here does not mean *mistake*; it is introduced into the results by external influences beyond the control of the experimenter. The static heard when listening to a radio with poor reception is generally random; the discovery of the 3K microwave background radiation by Arno Penzias and Robert Wilson in 1965 resulted from *systematic* error (i.e., no matter what they tried, they could not eradicate it), and this in turn resulted in confirmation of a prediction by the scientist George Gamow (and others) in 1948. Systematic errors can lead to new discoveries! Probabilistic or stochastic models incorporate a measure of uncertainty, and, for example, predict the *probability* that the tumor will have $N(t)$ cells at time t. Furthermore, if the cell population of interest is rather small, a typical change in the number of cells may well be a significant fraction of the total population, and so the "state" of the population must be represented, say, by an integer-valued random variable.

It has already been noted that mathematical models are not "right" (though they may be wrong as a result of our ignoring fundamental processes), but they can be useful. One model may be better than another in that it has better explanatory features, or more specific predictions can be made that are subsequently confirmed, at least to some degree. Some of the models presented in this book are still controversial; in particular, the reaction-diffusion models of pattern formation presented in Chapter 12 are not universally accepted by biomathematicians. There are other models, for example, in the study of wound healing, which utilize more of the mechanical properties of the medium (skin in this case) and are therefore designated *mechanochemical* models (they are not discussed in this book). There is some disagreement about what types of climate models are the most appropriate and reliable for predicting changing climate patterns. Indeed, we know that all mathematical models are flawed to some extent: many by virtue of inappropriate assumptions made in formulating the model, or (which may amount to the same thing) by the omission of certain terms in the governing equations, or even by misinterpretation of the mathematical conclusions in the original context of the problem. Occasionally models may be incorrect because of errors in the mathematical analysis, even if the underlying assumptions are valid. And paradoxically it can happen that even a less accurate model is preferable to a more mathematically sophisticated one; it was the mathematical statistician John Tukey who stated that *"it is better to have an approximate answer to the right question than an exact answer to the wrong one."* This is well illustrated by Lee and Fraser (2001) in their comparison of the less accurate Airy theory of the rainbow (see Chapter 6) with the more general and powerful *Mie theory*. They write

> Our point here is not that the exact Mie theory describes the natural rainbow inadequately, but rather that the approximate Airy theory can describe it quite well. Thus the supposedly outmoded Airy theory generates a more natural-looking map of real rainbow colors than Mie theory does, even though Airy theory makes substantial errors in describing the scattering of

> monochromatic light by isolated small drops. *As in many hierarchies of scientific models, the virtues of a simpler theory can, under the right circumstances, outweigh its vices.*

In many calculus books one can find examples that illustrate well the importance of clear assumptions when modeling: tumor growth, for example (see pp. 42–43 in Chapter 3). So readers may find helpful the following quotation from the above-mentioned book, *Wind Waves* (1965), by the oceanographer encountered earlier, Blair Kinsman. He identifies both the importance and the ubiquity of assumptions made in the process of mathematical modeling.

> In the derivation to follow, it will be *assumed* that the Earth is flat, the water is of constant density, that the Coriolis force is negligible, that the density of the air can be neglected compared with the density of water, that the body of water is of infinite extent and completely covered by waves, and that viscosity and surface tension can be neglected. Moreover, *to simplify the problem*, it will be *assumed* that there is no variation in the wave properties in the *y*-direction. A few more *assumptions* will turn up along the way in deriving the equations to be solved, but the equations will still be unsolvable in closed form as they will be nonlinear (emphasis mine).

It should be pointed out that in the majority of mathematical models that follow in this book, their assumptions and consequences will not be formally laid out as they might be in a formal textbook. Indeed, that organizational style is not generally used (in the author's experience) even in the modeling literature. In the next several chapters we will introduce some simple approaches to developing models of the natural world around us, especially in light of the changes we have witnessed since the first edition of this book over twenty years ago: (a) improved methods of estimation, (b) the spread of epidemics, and (c) climate change. And therefore it is necessary to re-emphasize that mathematical modeling—and forecasting in particular—is not an exact science.

Limitations of Mathematical Forecasting

Mathematical forecasting is an indispensable tool in both epidemiology and climate science, allowing us to simulate complex systems, anticipate future states, and inform policy decisions. However, these forecasts face profound limitations due to a combination of (a) extreme parameter sensitivities, (b) poor parameter identifiability, and (c) unreliable data. These limitations are especially pronounced in systems governed by nonlinear dynamics and feedback loops, which are common in both fields.

(i) Epidemiology

(A) EXTREME PARAMETER SENSITIVITIES

Epidemiological models, such as the classic SIR (Susceptible-Infected-Recovered) framework (discussed in Chapter 14) and its many extensions, are highly sensitive to key parameters such as the basic reproduction number (R_0), the incubation period, and contact rates. Small errors or changes in these values can lead to dramatically different outcomes, especially in the early stages of an outbreak. For example, an R_0 of 2.0 versus 2.5 may not seem like a large difference, but it can significantly alter predictions of total infections and healthcare burden due to the exponential nature of disease spread.

(B) POOR PARAMETER IDENTIFIABILITY

In practice, many parameters in epidemiological models cannot be uniquely determined from available data. For instance, distinguishing between a high transmission rate and a long infectious period may yield the same observed case curve. This leads to the issue of non-identifiability, where multiple parameter combinations fit the data equally well. This complicates efforts to make reliable forecasts or evaluate interventions, such as vaccination or social distancing.

(C) UNRELIABLE DATA

Epidemiological forecasting often relies on noisy, incomplete, or delayed data. Underreporting, changes in testing strategies, misclassification of cases, and inconsistent data collection across regions all introduce uncertainty. During the COVID-19 pandemic, for example, case counts, death tolls, and even hospitalization rates were inconsistently reported across jurisdictions, limiting the accuracy and comparability of model outputs. Additionally, asymptomatic carriers are often undetected, skewing prevalence estimates.

(ii) Climate Change

(A) EXTREME PARAMETER SENSITIVITIES

Climate models, especially general circulation models (GCMs), involve numerous interacting components—atmosphere, ocean, land, and ice. Some of these components exhibit tipping points or nonlinear feedbacks, such as ice-albedo feedback or permafrost carbon release, where small parameter changes can push the system into a qualitatively different state (e.g., accelerated warming). Moreover, projections are very sensitive to assumptions about climate sensitivity, defined as the temperature change associated with a doubling of CO_2 levels. A slight variation in this value can significantly alter long-term projections.

(B) POOR PARAMETER IDENTIFIABILITY

Climate models often contain subgrid-scale processes (e.g., cloud formation, ocean mixing) that are too complex or small to resolve directly and must be parameterized. These parameterizations involve uncertain coefficients that cannot be uniquely calibrated against global observations. In many cases, different sets of parameter values can yield similar historical temperature records, complicating efforts to project future trends. This equifinality limits confidence in long-term predictions.

(C) UNRELIABLE DATA

While the climate system benefits from a longer observational record than epidemiology, the data still suffer from heterogeneity in quality, coverage, and temporal resolution. Paleoclimate reconstructions involve indirect proxies (e.g., ice cores, tree rings) with intrinsic uncertainties. Even modern satellite and ocean measurements face calibration drift, gaps in coverage, or limited historical baselines. Additionally, socioeconomic scenarios (e.g., future emissions, land-use changes) inject further uncertainty, since they are not scientific parameters but plausible assumptions about human behavior.

Conclusion

In both epidemiology and climate science, forecasting is constrained not only by the mathematical models themselves but also by the uncertain, incomplete, and noisy nature of the inputs. The interplay of sensitive dependence on parameters, nonuniqueness of parameter estimation, and unreliable data means that forecasts must be interpreted probabilistically rather than deterministically. Sensitivity analysis, ensemble modeling, and data assimilation techniques are essential tools to mitigate these limitations, but fundamental uncertainties remain irreducible, especially over long timescales or in systems near critical thresholds.

Potential Hazards for Real-Time Prediction

In June 2020 the journal *Nature* published a comment by Andrea Saltelli (and a great many other people) in which the authors suggest five ways to ensure that models serve society. Of course, the word 'models'—both deterministic and statistical—is used in a much broader sense than the mathematical models discussed here, but statistical and probabilistic analysis is a usually a major component of models in general. Indeed, when it comes to public health policy, reliable statistical analysis is crucial to the formulation of policy recommendations. They are summarized as follows:

1. **Mind the assumptions:** Models rely on assumptions that may not be valid across different situations or for new applications. It is crucial to assess the sensitivity of model outputs to these assumptions and consider whether they are appropriate for the specific context.
2. **Mind the hubris:** Overly complex models can become difficult to understand and manage, leading to reduced precision and usefulness. Simpler models can often be more effective and less prone to error than overly complicated ones.
3. **Mind the framing:** Models reflect the biases and interests of their developers, which can shape their purpose and application. It's important to match the model's framing to its intended use to ensure it serves its purpose effectively.
4. **Mind the consequences:** Quantification can sometimes backfire, creating a false sense of certainty or obscuring critical factors. Models should provide full and frank disclosure of their strengths and limitations to avoid generating misleading outcomes.
5. **Mind the unknowns:** Models are inherently limited by the factors they do not account for, and it is important to acknowledge this ignorance. Instead of providing a single, overly confident answer, models should convey the uncertainty associated with their predictions.

The recommendations in the article are further summarized in a series of succinct statements, which are quoted here:

> Complexity can be the enemy of relevance.
>
> Excessive regard for producing numbers can push a discipline away from being roughly right toward being precisely wrong.
>
> Mathematical models are a great way to explore questions. They are also a dangerous way to assert answers.
>
> Following these five points will help to preserve mathematical modelling as a valuable tool. Each contributes to the overarching goal of billboarding the strengths and limits of model outputs. Ignore the five, and model predictions become Trojan horses for unstated interests and values. Model responsibly.

With such warnings and provisos in mind, the aim of this book is *not* to present *thorough* mathematical descriptions of many naturally occurring phenomena—an impossible task—but instead to try and present a compilation and synthesis of several mathematical models that have been developed within these contexts. The extensive set of references in the bibliography is provided to encourage the inquisitive reader to pursue the original articles and books in which these models were first presented. There are undoubtedly many published papers on these topics of which I am unaware, and that certainly would have enhanced this book, and for that I must point out, regrettably, that one has to stop somewhere (the publisher requires it).

The Benefits of Forecasting

Nevertheless, honest forecasting (i.e., data-driven)—whether in pandemics or climate change—remains enormously valuable despite the recognized limitations of mathematical models. Forecasts, when framed transparently with their assumptions, uncertainties, and potential error margins clearly identified, provide society with structured guidance for planning and preparedness. In pandemics, even imperfect models help policymakers anticipate surges in healthcare demand, evaluate the consequences of interventions (like distancing or vaccination), and avoid being blindsided. In climate science, models have consistently highlighted long-term warming trends, sea-level rise, and regional shifts in weather extremes, guiding infrastructure planning, international agreements, and adaptation strategies. The key lies not in demanding precise predictions, but in understanding ranges of possible futures and their implications.

Hindsight underscores the benefits of such foresight. Looking back, early epidemic models (for HIV, SARS, and COVID-19) may have been imperfect in details but were invaluable in highlighting exponential growth risks and the importance of timely interventions. Similarly, climate projections from decades ago—though coarse by today's standards–have largely captured the observed trajectory of global warming, validating the scientific warnings. In both domains, hindsight shows that societies that acted on forecasts, even cautious or conservative ones, were often better prepared than those that completely ignored them. Thus, the greatest value of honest forecasting lies not in precision, but in cultivating foresight, resilience, and the willingness to act before hindsight forces us to. It seems appropriate to quote environmental mathematician Martin Walter here (and again in Chapter 3):

> But humans fooling other humans is quite common. When humans are dealing with Nature, however, honesty is imperative; for Nature cannot be fooled.

(continued...)

INDEX