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# Introduction

*Philosophy is written in this enormous book which is continually open before our eyes (I mean the universe), but it cannot be understood unless one first understands the language and recognizes the characters with which it is written. It is written in a mathematical language, and its characters are triangles, circles, and other geometric figures. Without knowledge of this medium it is impossible to understand a single word of it; without this knowledge it is like wandering hopelessly through a dark labyrinth.*

—Galileo, *The Assayer*, as quoted in Italo Calvino's *Why Read the Classics?*

This book is the result of my decades of teaching mathematics. The text attempts to introduce science and engineering students to the long history that underpins the concepts that are currently used in mathematics in an often unconscious, mechanical way. The symbols developed to represent mathematical concepts also have a long history. Since the time of Galileo, it has been postulated that the natural world is written in the language of mathematics. However, we rarely delve into the history of this science, which represents a double loss—a loss that diminishes both cultural enrichment and intellectual understanding. If we know where the concepts come from and what arguments led to their first formulation, we can be better prepared to use them as part of our mathematical toolbox.

This book is divided into nine chapters with fifty-four sections in total. Each section examines just one or two mathematical symbols, their history, and any variants they may have generated

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over time. Each section is self-contained, so they can be read in any order. The book is designed so that the reader can proceed from one topic to the next, exploring the origins of our mathematical language by following the inspiration of the moment. In my experience, these stories can also serve to awaken students' enthusiasm for math, providing them with intellectual stimuli when they start to get bored or are tempted to give up in the face of the formalism on the blackboard. It is always interesting to hear about the mathematics of Leibniz or Gauss or to see how many different typographical crosses we have adopted as mathematical symbols.

Presenting the book in self-contained sections has the associated effect of creating some redundancy. Some explanations, or the presentation of a particular mathematician, may appear in two or more sections of the text. I have tried to limit repetitions to the minimum possible but have not avoided them altogether. I appeal to the readers' patience, because repetition may help to remember.

My first seminar on the history of mathematical symbols was held in Berlin in 1997. The topics gathered here were scribbled down over time, sometimes in English and sometimes in German. However, for years, I could not find the right style to develop the topics into a coherent book. So I went back to my mother tongue, Spanish, and in 2017, the manuscript found its final, more fluid, and more enjoyable form. From this more comfortable medium of exposition, it has now been easier to prepare an English version of the work.

This book is not an encyclopedic treatise, like Florian Cajori's monumental 1928 work, *A History of Mathematical Notations*, which, to this day, has not been surpassed. Nor is it about covering all mathematical notations over time, one by one, author by author, century by century. Rather, it is about marveling at

the history of mathematical endeavor and getting to know the giants on whose shoulders we stand today. It is about understanding how the language of mathematics was forged through a collective effort spanning more than twenty centuries and many bygone empires. What remains, the only constant, is the progress of mathematics—always in search of a better way to express relationships among abstract concepts, always in search of its own voice.

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